tory explanation has been found for the profound effect of such a small amount of gas or the behavior of the tube.

The investigation is being continued and a more detailed account will be published in the near future.

[1] Barkhausen and Kurz, *Phys. Zs., Leipzig*, **21**, No. 1, Jan. 1920, p. 1.
[2] Whiddington, *Radio Review*, Nov. 1919, p. 53.
[3] Gill and Morrell, *Phil. Mag.*, **44**, No. 259, July 1922, p. 161.

THE REFRACTION OF X-RAYS IN CALCITE

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The effect of refraction in X-ray spectra has been discussed by Stenström,[1] who made some determinations of the index of refraction from the relative displacement of the several orders. In most cases the effect was too small to admit of measurement, but for sugar and gypsum crystals he obtained some values for wave-lengths greater than 2.5 Å.

The present paper applies a modification of the same method to the reflection from calcite of the $K\alpha_1$ line of Molybedenum, 70783 Å.

Since we measure the angle from the crystal face, and not the normal, the customary equation for the index of refraction becomes

$$\nu = \frac{\cos\theta}{\cos\theta'} \qquad (1)$$

where θ is the glancing angle outside the crystal and θ' the angle of the beam inside. We will use a subscript to indicate orders higher than the first. Placing $\nu = 1 - \delta$, Stenström computes the values of δ from the equation

$$\delta = \frac{\left(\frac{\sin\theta_m}{m}\right)^2 - \left(\frac{\sin\theta_n}{n}\right)^2}{2\left(\frac{\cos\theta_m}{m}\right)^2 - 2\left(\frac{\cos\theta_n}{n}\right)}$$

where m and n are any two orders.

As the shift is small in any case, it seemed desirable to express the value of δ directly in terms of angle, in order to more readily determine the effect of errors of observation which are liable to be of the same order of magnitude as δ itself. Now from (1) we have

$$\sin^2\theta' = 1 - \nu^{-2}\cos^2\theta$$

Since δ is small, we write

$$\nu^{-2} = 1 + 2\delta$$

whence

$$\sin\theta = [1 - \cos^2\theta\,(1 + 2\delta)]^{1/2} = [\sin^2\theta - 2\delta\cos^2\theta]^{1/2}$$

and again neglecting terms containing higher powers of δ.

$$\sin\theta' = \theta - \delta\,\frac{\cos^2\theta}{\sin\theta} \tag{2}$$

Taking arc sin of each side, we have since δ is small,

$$\theta' = \theta - \delta\,\frac{\cos^2\theta}{\sin\theta}\cdot\frac{1}{\cos\theta} \tag{3}$$

$$\theta' = \theta - \delta\cot\theta$$

Now in this equation, θ is the crystal angle measured, hence by determining the angles of any characteristic line for several orders, it will be possible to determine δ, or at least to find the limits of its magnitude. The angles being measured, those of higher orders are reduced to the equivalent first order angle, so that all angles may be compared directly. The proportion that each is affected by refraction being known from (3), the absolute value of δ is readily determined.

Instead of using (3), it is possible to substitute (2) in the relation

$$n\lambda = 2d\sin\theta_n'$$

obtaining, since δ is small

$$\frac{n\lambda}{2\sin\theta_n} = d\,(1 - \delta\cot^2\theta_n) \tag{4}$$

thus determining δ from the increase in apparent grating space with order. However, since the limits of error have to be carried through in the computations, the previous method seems to be more convenient.

The spectrometer used has been described elsewhere.[2] The tube was of the water cooled type with a molybdenum target. Measurements of θ were made for the first three orders of the $K\alpha_1$ line, using a crystal of clear Iceland spar. The measurements were gone over several times and it was found that the angles could be repeated to within 15″. This is just about the limit of accuracy of the instrument which was determined by other methods to be about 20″.

The following mean values were obtained.

$$\theta = 6^\circ\ 42'\ 43'' \pm 10''$$
$$\theta_2 = 13^\circ\ 30'\ 45'' \pm 10''$$
$$\theta_3 = 20^\circ\ 31'\ 22'' \pm 10''$$

where the additional terms represent the limits of error.

It should be remarked that if the crystal is not mounted so that the mean reflecting plane is directly over the center of rotation, and the face of the crystal be slightly curved, a shift of the lines will be obtained, that might be taken for refraction. To eliminate this possibility, the crystal was mounted on a slide with a micrometer screw attached, so that it might be moved in a direction perpendicular to its face. The crystal being mounted as nearly as possible over the center of rotation by mechanical methods, the adjustment was changed by small fractions of a millimeter, reading each time on both sides the crystal and chamber angles of the α_1 line. If curvature is present, the crystal angle will change as the crystal is moved. Twice the crystal angle is plotted against the readings of the micrometer head and on the same sheet, the chamber angle is also plotted against these readings. The intersection of the two lines gives the proper micrometer setting, since the crystal must be over the center, when the chamber angle is exactly twice the crystal angle. With the crystal used, it was found that the curvature was very slight, a shift of less than one second of arc being found within the range of adjustment. This was checked for each order.

We proceed then to reduce the angles measured to the corresponding first order angles. Now $\theta_n = \theta_n' + \Delta$ where Δ includes the slight increase due to refraction and also the error of observation. Since Δ is small

$$\sin \theta_n = \sin \theta_n' + \Delta \cos \theta_n$$

$$\frac{\sin \theta_n}{n} = \sin \theta_n' + \Delta \frac{\cos \theta_n}{n}$$

$$\text{arc sin} \left(\frac{\sin \theta_n}{n}\right) = \theta' + \Delta \frac{\cos \theta_n}{n \cos \theta}$$

$$\theta' = \text{arc sin} \left(\frac{\sin \theta_n}{n}\right) - \Delta \frac{\cos \theta_n}{n \cos \theta}$$

and the coefficient of Δ can be computed for each order, it being .49 for the second and .31 for the third. Multiplying the errror limits by these coefficients and introducing the refraction from (3) which is also multiplied by them, we have the following values for θ':

$$\begin{aligned} \theta' &= 6^\circ\ 42'\ 43'' \pm 10'' - 1''.76 \cdot \delta \cdot 10^6 \\ &= 6^\circ\ 42'\ 33'' \pm 5'' - 0''.63 \cdot \delta \cdot 10^6 \\ &= 6^\circ\ 42'\ 38''.5 \pm 3'' - 0''.27 \cdot \delta \cdot 10^6 \end{aligned}$$

and we have to determine δ from these equations.

Plotting the values of the angles against the order, using lines covering the limits of error instead of points to locate the extremities of the ordinates,

we have to pass through these lines a curve whose ordinates are proportional to the coefficients of δ. Now the values found do not lie well on any curve of this type, the second order being entirely too low. The curves that fitted most nearly gave a value of $\delta = 3 \times 10^{-6}$ while the error limits would permit of values ranging from zero to several times this. It will be seen that this corresponds to a shift of the first order of 5″, so that for this wave-length, the effect of refraction is very slight.

This value $\delta = 3 \times 10^{-6}$ necessarily contains large possible errors. It is, however, of the same order of magnitude as that obtained by the method of total reflection.

An independent determination of δ was obtained by measuring the angle of total reflection as suggested by the experiments of A. H. Compton[3] on total reflection from glass and lead. The ionization chamber was shielded from the direct beam and the spectrum searched at angles close to grazing incidence. Total reflection appeared as a line, approximately 4′ wide at half maximum, slightly unsymmetrical and of an intensity about 25 times the surrounding values which coincided with the leak. To test if this line consisted principally of the monochromatic radiation corresponding to the $K\alpha$ line, a zirconium screen was introduced. This diminished the intensity without materially changing the shape of the line or shifting its position.

Measurements were made on each side of the zero position, and the mean of the readings gave 6′ 30″, the value being good to 30″. Now for total reflection, putting $\theta' = 0$ in (1) we have

$$\nu = \cos\theta \text{ or } \delta = \frac{\sin^2\theta}{2}$$

whence

$$\delta = 1.7 \pm .5 \times 10^{-6}$$

Comparing this with the theoretical value from the Lorentz equation

$$\delta = \frac{n e^2}{2\pi m} \cdot \frac{1}{\nu}$$

we take the density of calcite as 2.71 giving 81.5×10^{22} electrons per cubic centimeter, and place ν equal to the frequency of the molybdenum $K\alpha$ line, whence $\delta = 1.85 \times 10^{-6}$.

Determinations of the total reflection made with a second crystal gave results which checked with the preceding. Incidently it developed that the face of this second crystal had been ground at a slight angle to the reflecting planes, the crystal zero determined by the angle of total reflection differing from that obtained from the line spectra by an angle of 44′. This suggests an extension of the method of determining the refraction

from the shift of the spectral lines, for if the face of the crystal were ground at the proper angle to the reflecting planes, the shift would be increased enough to make accurate determinations possible.

[1] *Experimentelle Untersuchungen der Röntgenspektra*, Lund 1919.
[2] *J. Amer. Opt. Soc.*, May 1922.
[3] *Physic Rev., Ithaca*, July 1922.

THE COMPRESSIBILITY OF METALS AT HIGH PRESSURES

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This note briefly summarizes results which will be found in full detail in a forthcoming number of the PROCEEDINGS of the American Academy of Arts and Sciences. A considerable part of the expenses of the experiment was defrayed by a generous appropriation from the Rumford Fund of the American Academy.

A new method has been devised by which the difference of the linear compressibility of a solid in the form of a rod or wire and the linear compressibility of iron may be measured with an accuracy high enough to give the variation of compressibility with pressure or temperature. In addition, I have improved the method which I previously used[1] for the measurement of the absolute linear compressibility of iron, so that I have now measured the change of linear compressibility with pressure with some accuracy, and I have also obtained improved values for the change of linear compressibility with temperature. By combining these absolute measurements on iron with the measurements of the difference of compressibility the absolute linear compressibility of a number of metals has been obtained.

If the metal is equally compressible in all directions, the absolute volume compressibility may be calculated from the linear compressibility. The great majority of metals crystallize in the cubic system or in the hexagonal close packed arrangement of spheres, and for these the compressibility is equal in all directions. Table I gives the compressibility, χ (defined as $(\partial v/\partial p)_T$ where v is the volume of the amount of the metal that under standard conditions occupies 1 cc.) at atmospheric pressure at 30°, the pressure derivative of the compressibility, and the change of compressibility per degree rise of temperature for a number of such metals. The unit of pressure is the kilogram per square centimeter, the pressure range of the experiments was 12000 kg./cm.^{2}, and measure-